$$-\frac{1}{k_0^2}\nabla\times\nabla\times\bar{E}+\frac{1}{k_1^2U}\nabla\nabla\cdot\bar{E}+\left(1-\frac{X}{U}\right)\bar{E}+\frac{i\bar{Y}}{U}\times\left(\bar{E}-\frac{1}{k_0^2}\nabla\times\nabla\times\bar{E}\right)=\frac{1}{k_1^2U}\cdot\left(\frac{1}{\gamma}\frac{\nabla N_0}{N_0}-\frac{\gamma-1}{\gamma}\frac{\nabla T_0}{T_0}\right)(\nabla\cdot\bar{E})+\frac{1}{U}\nabla\left\{\frac{1}{k_1^2}\left(\frac{\gamma-1}{\gamma}\frac{\nabla N_0}{N_0}-\frac{1}{\gamma}\frac{\nabla T_0}{T_0}\right)\cdot\left(\bar{E}-\frac{1}{k_0^2}\nabla\times\nabla\times\bar{E}\right)\right\} \quad (2)$$

where $\bar{Y}(\bar{r})=(e\mu/\omega m)\,\bar{H}_0(\bar{r})$, $\bar{H}_0(\bar{r})$ being the inhomogeneous magnetostatic field, and $U(\bar{r})=1-iZ(\bar{r})=1-i\,\nu(\bar{r})/\omega$, $\nu(\bar{r})$ being the average collision frequency, and the plasma is inhomogeneous both in average density $N_0(\bar{r})$ and in average temperature $T_0(\bar{r})$.

For the particular case under consideration by Lonngren[1] one has $U=1$, $\bar{Y}=0$, $\nabla T_0=0$, (T_0=constant), and $k_1=\omega/a$=constant, and (2) becomes:

$$-\nabla\times\nabla\times\bar{E}+\frac{a^2}{c^2}\nabla\nabla\cdot\bar{E}+k_0^2(1-X)\bar{E}=\frac{a^2}{\gamma c^2}\frac{\nabla N_0}{N_0}\nabla\cdot\bar{E}+\frac{a^2}{c^2}\frac{\gamma-1}{\gamma}\Delta\left\{\frac{\nabla N_0}{N_0}\cdot\left[\bar{E}-\frac{1}{k_0^2}\nabla\times\nabla\times\bar{E}\right]\right\}\cdot \quad (3)$$

For small inhomogeneities $|\nabla N_0/N_0|\ll 1$, (1) and (3) are identical, but they are not identical in general.

The reason for the disagreement between (1) and (3) seems to be the suggestion by Lonngren[1] to use the equation of state

$$p_1=\gamma K T_0 N_1 \quad (4)$$

for the inhomogeneous plasma. The equation of state for the adiabatic process and no viscous effects has been given by Unz[3] in the form:

$$\frac{Dp}{Dt}=\gamma K T\frac{DN}{Dt}\cdot \quad (5a)$$

Using small signal theory in (5a) for inhomogeneous nondrifting plasma, it has been pointed out by Rao and Unz[4] that the adiabatic equation of state becomes for the present case:

$$\frac{\partial p_1}{\partial t}+\bar{u}_1\cdot\nabla p_0=\gamma K T_0\left[\frac{\partial N_1}{\partial t}+\bar{u}_1\cdot\nabla N_0\right]. \quad (5b)$$

Only for the particular case of homogeneous plasma $\nabla N_0=\nabla p_0=0$ (5b) agrees with (4). However, for the inhomogeneous plasma one should use the equation of state (5b) rather than (4) as suggested by Lonngren,[1] and as a result one will obtain the wave equation (3) for the inhomogeneous warm plasma given by Unz[2] rather than (1) given by Lonngren.[1]

D. Kalluri
H. Unz
Dept. of Elec. Engrg.
University of Kansas
Lawrence, Kan. 66044

[3] H. Unz, "Wave propagation in drifting isotropic warm plasma," *Radio Sci.*, vol. 1, pp. 325–328, appendix A, March 1966.

[4] S. S. Rao and H. Unz, "On the adiabatic equation of state for inhomogeneous warm plasmas," *Proc. IEEE (Letters)*, vol. 54, pp. 1224–1225, September 1966.

Author's Reply[5]

Kalluri and Unz note that at the low-frequency limit of a collision dominated plasma, the pressure obeys an equation of state of the form[6]

$$\frac{d}{dt}(Pn^{-\gamma})=0. \quad (1)$$

This modifies our equation (12)[1] to

$$E_z=C_2\epsilon^{-[(\gamma-2)/\gamma][\dot{N}_0/N_0]z}\cos\left\{\frac{k_0}{\alpha}\left[1-\frac{\omega_p^2}{2\omega^2}\right]z+\frac{\alpha}{k_0}\frac{\gamma-1}{2\gamma}\left[\frac{\dot{N}_0}{N_0}-\left(\frac{\dot{N}_0}{N_0}\right)^2\right]z+C_1\right\}. \quad (2)$$

Acknowledgment

We wish to thank D. Kalluri and H. Unz for this comment and Profs. H. C. S. Hsuan and D. C. Montgomery for discussions concerning it.

K. E. Lonngren
Dept. of Elec. Engrg.
University of Iowa
Iowa City, Iowa

[5] Manuscript received January 3, 1967.

[6] D. C. Montgomery and D. A. Tidman, *Plasma Kinetic Theory*. New York: McGraw-Hill, 1964, pp. 194–209.

Properties of a TEM Transmission Line Used in Microwave Integrated Circuit Applications

Introduction

A somewhat unusual TEM transmission line configuration that is useful in certain microwave integrated circuit realizations[1,2] consists of a strip-line center conductor on a dielectric substrate coaxial with a cylindrical outer conductor, as shown in Fig. 1(a). The cylindrical outer conductor is perturbed slightly by slots that support the dielectric substrate. Due to the irregular conductor geometry and the presence of the dielectric substrate, exact closed-form theoretical solutions for the characteristic impedance Z_0 and the effective dielectric constant $\bar{\epsilon}$ of this transmission line have heretofore not been obtained. However, an approximate theoretical solution has been obtained using the slightly simplified cross section of Fig. 1(b), in which the outer conductor slots are neglected. In addition, the validity of this approximate solution has been demonstrated experimentally.

[1] H. C. Okean, "Integrated microwave tunnel diode devices," *1966 G-MTT Symp. Digest*, pp. 135–140, May 1966.

[2] H. C. Okean, "Microwave amplifiers employing integrated tunnel diode devices," to be published.

Theoretical Results

The effective dielectric constant $\bar{\epsilon}$ and the characteristic impedance Z_0 of the configuration of Fig. 1(b) are obtained from its static, unit-length interconductor capacitance, $C(pF/in)$, using the fundamental relationships[3]

$$\bar{\epsilon}=\frac{C}{C_0},\qquad Z_0=\frac{84.9}{C_0(pF/in)\sqrt{\bar{\epsilon}}},\qquad C_0=C|_{\bar{\epsilon}=1}. \quad (1)$$

To calculate C in this case in which its direct solution for the given cross section does not exist, we locate the latter in the complex ϕ plane [Fig. 1(b)], and use the property that C is invariant under conformal transformations $t=t(\phi)$ of the ϕ plane.[3] We then select t to yield that transformed cross section for which C can most conveniently be calculated directly. In this case, we choose the familiar bilinear transformation

$$t=\frac{1+\phi}{1-\phi}=u+jv;\quad \phi=\mu+j\psi \quad (2)$$

which yields the transformed t-plane cross section with new dimensions, as shown in Fig. 1(c). C cannot be calculated directly in the t-plane, nor is there any convenient conformal transformation of the t-plane to yield a configuration amenable to exact calculation of C. However, if we approximate the t-plane cross section by the t'-plane cross section of Fig. 1(d) (valid particularly for small arg t and hence, for small substrate thickness 2Δ, and exact in the limit $\Delta=0$ or $\epsilon=1$) and use the method of images,[3] the resulting configuration can be solved directly for $\tilde{C}\approx C$. In specific, we define C' as the coupled-strip capacity between the transformed center conductor and its image. Then the desired C, approximated by $\tilde{C}$, the capacity between the center conductor and the imaginary-axis ground conductor, is given by[4]

$$C\approx\tilde{C}=2C'=0.45\left[\frac{K(k)}{K(k')}+\frac{1}{2}(\epsilon-1)\frac{K(k_1)}{K(k_1')}\right]pF/in \quad (3)$$

where

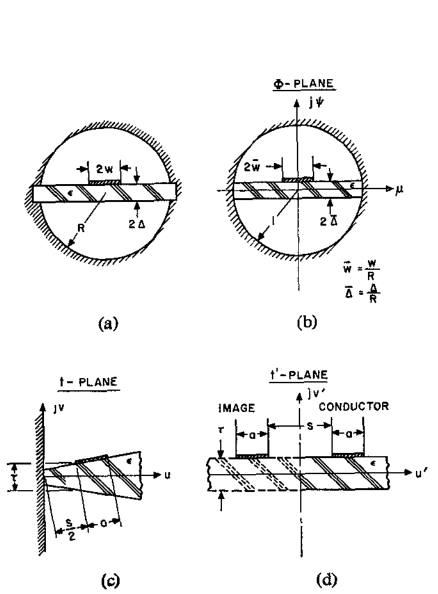
$$K(\alpha)=\int_0^{\pi/2}\frac{d\theta}{\sqrt{1-\alpha^2\sin^2\theta}}$$

$$k'=\sqrt{1-k^2}=\frac{s}{2a+s}$$

$$k_1'=\sqrt{1-k_1^2}=\frac{\tanh\left(\frac{\pi}{4}\frac{s}{\tau}\right)}{\tanh\left(\frac{\pi}{4}\frac{2a+s}{\tau}\right)}.$$

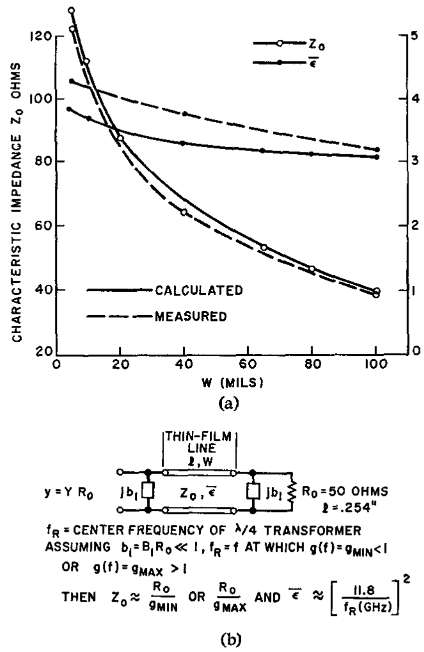
[3] R. E. Collin, *Field Theory of Guided Waves*. New York: McGraw-Hill, 1960, pp. 119–169.

[4] R. F. Frazita, "Transmission line properties of coplanar parallel strips on a dielectric sheet," M.S. thesis, Polytechnic Institute of Brooklyn, New York, 1965.



(a) Cross section of TEM transmission line with strip center conductor and cylindrical outer conductor geometry.
(b) Simplified cross section of TEM line with strip center conductor and cylindrical outer conductor.
(c) Transformed t-plane TEM line cross section.
(d) Approximate t-plane TEM line cross section.

Fig. 1. TEM transmission line geometry.



(a) Calculated and measured Z_0 and $\bar{\epsilon}$ for a given TEM transmission line as function of center conductor width.
(b) Determination of Z_0 and $\bar{\epsilon}$ from measured data.

Fig. 2. Calculated and measured TEM transmission line parameters.

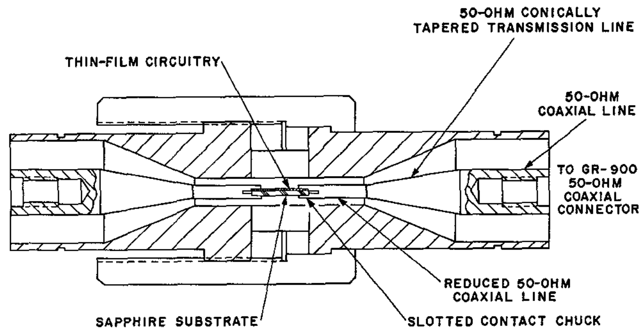


Fig. 3. Two-port coaxial test fixture for transmission line measurements.

The normalized, transformed dimensions s, a, τ are given in terms of original dimensions, $\bar{w}$, $\bar{\Delta}$ (normalized to outer conductor radius), using (2) by:

$$s = [2|t|]_{\mu=-\bar{w}} = 2\sqrt{\frac{(1-\bar{w})^2+\bar{\Delta}^2}{(1+\bar{w})^2+\bar{\Delta}^2}} \quad (4a)$$

$$a = |t|_{\mu=\bar{w}} - |t|_{\mu=-\bar{w}} = \sqrt{\frac{(1+\bar{w})^2+\bar{\Delta}^2}{(1-\bar{w})^2+\bar{\Delta}^2}} - \sqrt{\frac{(1-\bar{w})^2+\bar{\Delta}^2}{(1+\bar{w})^2+\Delta^2}} \quad (4b)$$

$$\tau = [2\mathrm{Im}(t)]_{\mu=0} = \frac{4\bar{\Delta}}{1+\Delta^2}. \quad (4c)$$

Substituting (4) and (3) in (1) for a particular[1,2] sapphire substrate and cylindrical outer conductor ($\bar{\Delta}\approx 0.4$, $\epsilon=9$, $R=0.149$ inch), Z_0 and $\bar{\epsilon}$ are calculated as functions of center conductor width w over the range 0.005 to 0.100 inch ($\bar{w}=0.0335$ to 0.670) as plotted in Fig. 2(a).

Experimental Results

Experimental verification of this theory is obtained by the following approximate method of determining Z_0 and $\bar{\epsilon}$. A series of 0.254 in. length of thin-film line of varying widths w deposited on identical sapphire substrates are each mounted in the two-port coaxial test fixture shown in Fig. 3. The resulting transmission line cross section is that of Fig. 1(a) with the dimensions as previously quoted.

In this test mount, the substrate rests in slots in the 0.150 inch diameter cylindrical outer conductor surrounding it and is held in place by slotted contact chucks in the 0.065 inch diameter cylindrical center conductors that contact the thin-film circuit at both ends of the substrate. Conically tapered coaxial transmission line sections, designed to maintain a 50-ohm characteristic impedance[5,6] provide well-matched transitions (VSWR < 1.10 with respect to a 50-ohm termination) over 2.0 to 7.5 GHz, between a 50-ohm coaxial line midsection inserted between the substrate contact chucks and the 50-ohm input and output GR-900 precision connectors. The total measured mismatch due to the pair of coaxial thin-film interfaces and the tapers exhibits a VSWR < 1.15 over the 2.0 to 7.5 GHz range.

The measurement procedure consists of obtaining, over a wide frequency range, the input admittance locus at one port of the test fixture, referenced to the coaxial thin-film interface (Fig. 3), with the other port terminated in 50 ohms. At some frequency f_R, the length of thin-film line behaves as a quarter-wave impedance transformer, permitting approximate calculations of Z_0 and $\bar{\epsilon}$ from the measured data, as shown in Fig. 2(b).

The resulting experimental values of Z_0 and $\bar{\epsilon}$ plotted as functions of w along with their theoretical counterparts in Fig. 2(a) show reasonable agreement with the theory. The empirical 50-ohm center conductor ($w=0.065$ inch) results in an input VSWR of less than 1.15 over 2.0 to 7.5 GHz and yields an $\bar{\epsilon}=3.5$ when mounted in the 50-ohm terminated structure of Fig. 3.

Conclusions

Approximate theoretical solutions for the characteristic impedance and effective dielectric constant have been obtained for the transmission line configuration consisting of a strip-line center conductor on a dielectric substrate coaxial with a cylindrical outer conductor. The validity of these solutions has been demonstrated by experiment.

Acknowledgment

The author gratefully acknowledges the contributions of B. A. Unger for providing the thin-film transmission lines, E. C. Maslankowski for providing the test fixture, and R. H. Minetti for making the required measurements. In addition, he would like to thank W. J. Bertram for many helpful suggestions.

H. C. Okean[7]
Airborne Instruments Lab.
A Div. of Cutler-Hammer, Inc.
Melville, N. Y.

[5] Meinke and Gundlach, *Taschenbuch der Hochfrequenztechnik*. Berlin: Springer-Verlag, 1956, pp. 266–269.

[6] S. A. Schelkunoff, *Electromagnetic Waves*. Princeton, N. J.: Van Nostrand, 1943, pp. 287–290.

[7] Formerly with Bell Telephone Labs., Inc., Murray Hill, N. J.